CONIQUES

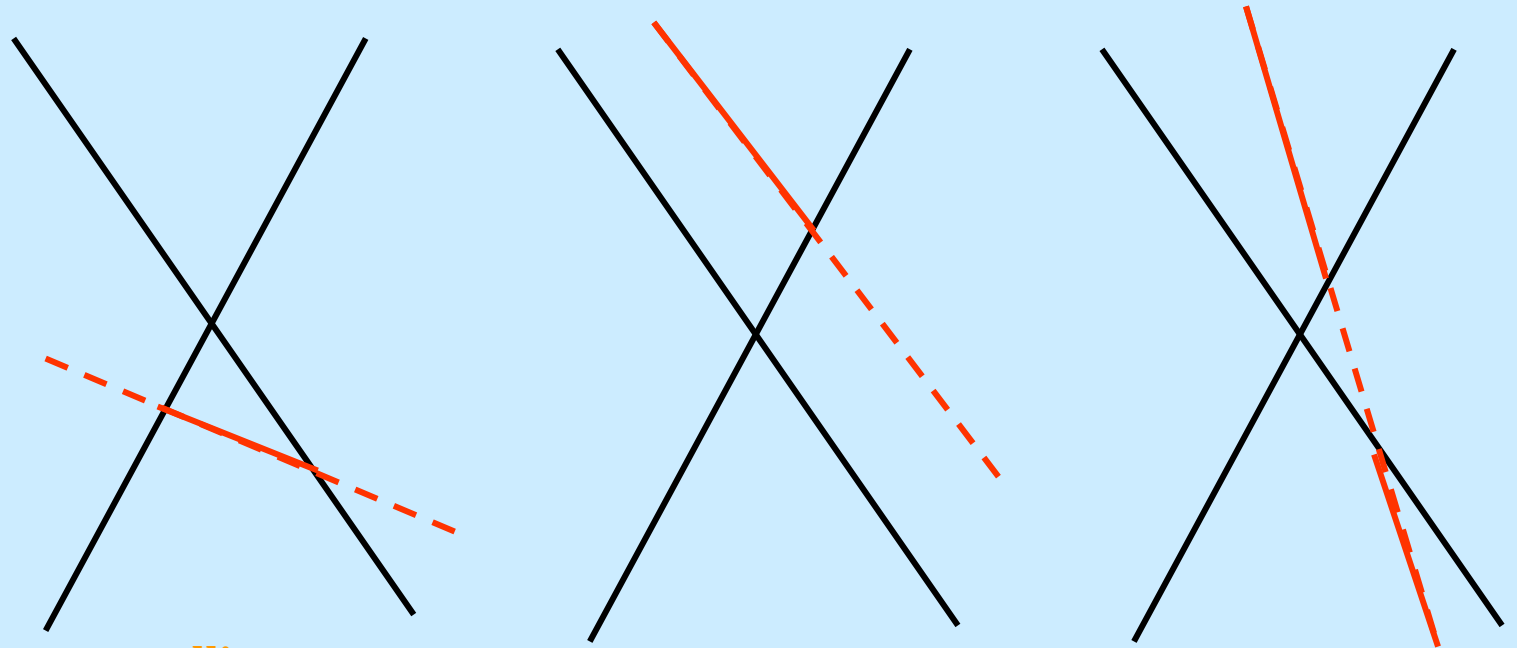
1. Définition géométrique dans l'espace

Intersections d'un cône de révolution avec un plan, d'où le nom

ellipse : le plan coupe une seule nappe du cône.

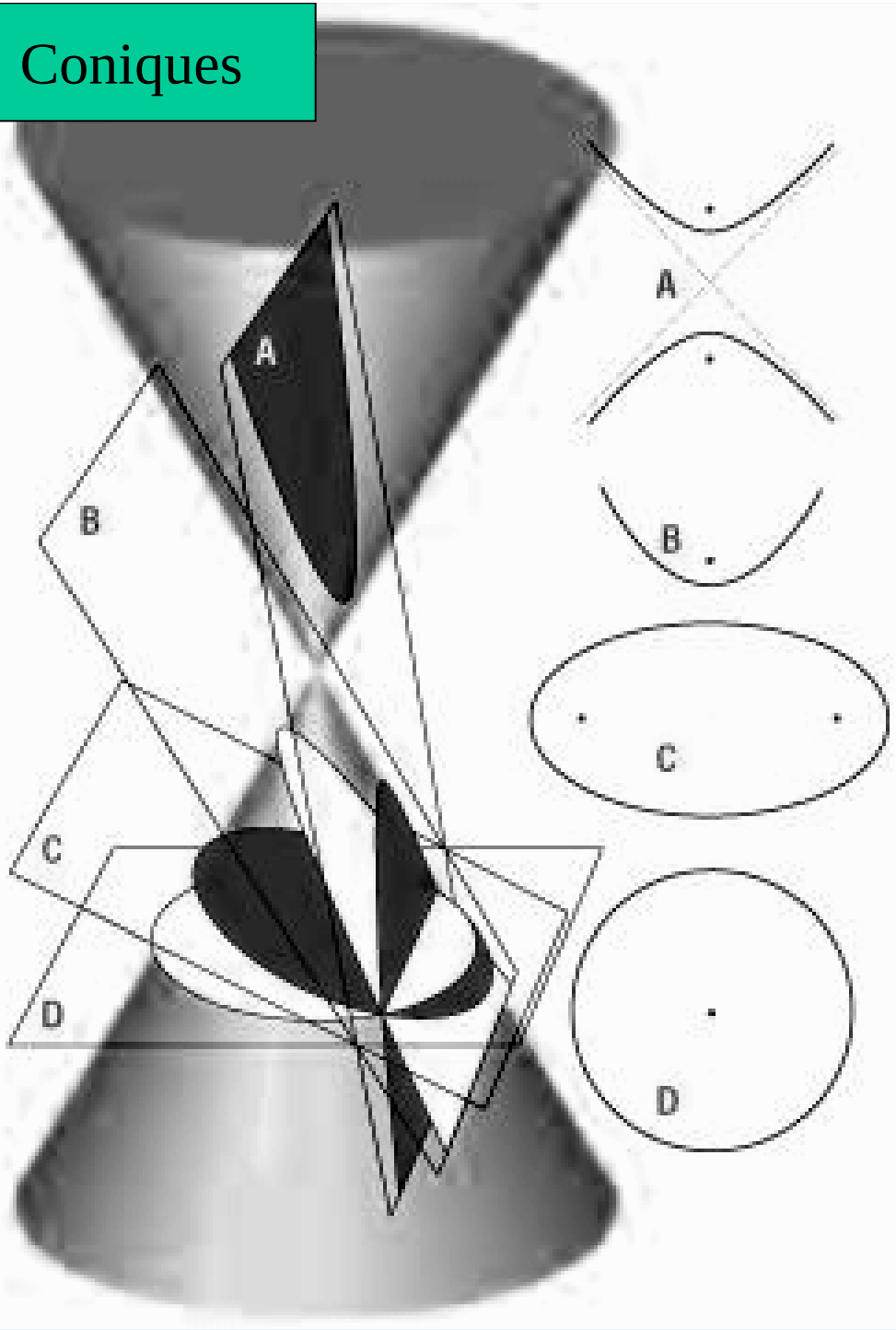
parabole : le plan coupe une seule nappe du cône en étant parallèle à une génératrice.

hyperbole : le plan coupe les deux nappes du cône.



ellipse **parabole** **hyperbole**

Coniques



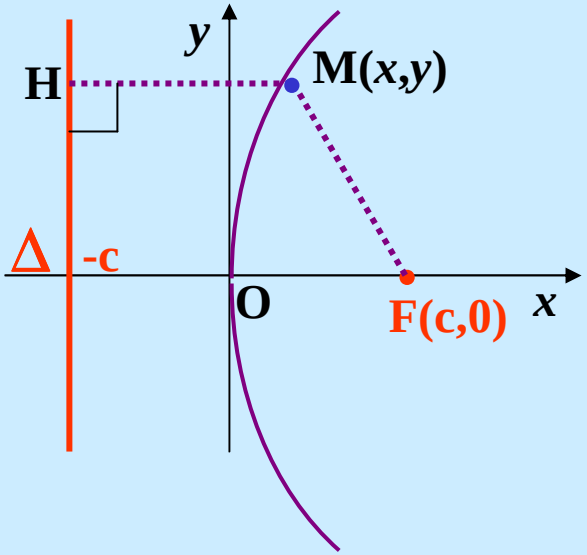
hyperbole

parabole

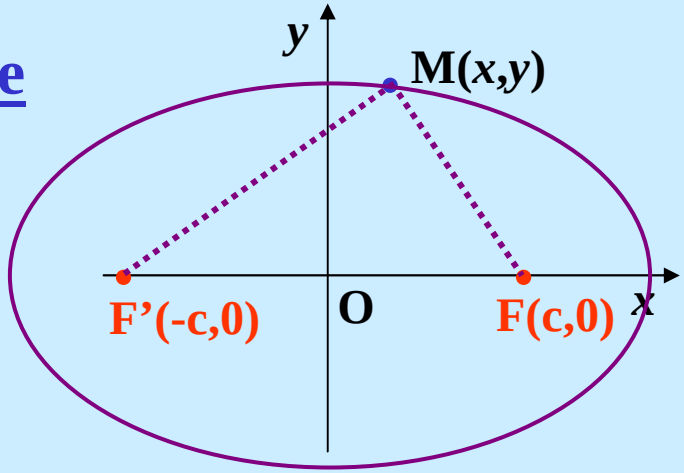
ellipse

cercle

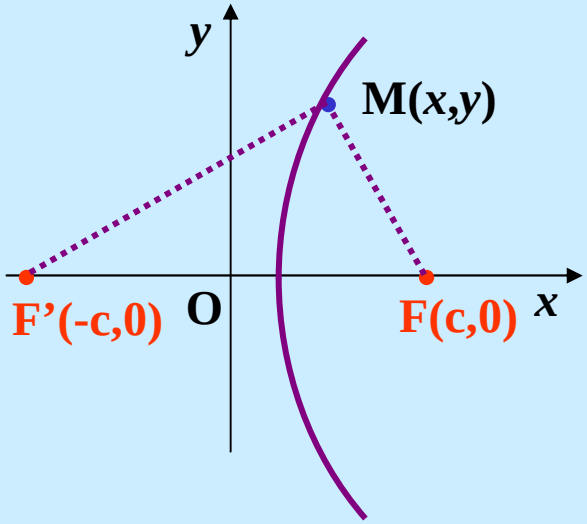
2. Définition géométrique plane



parabole



ellipse



hyperbole

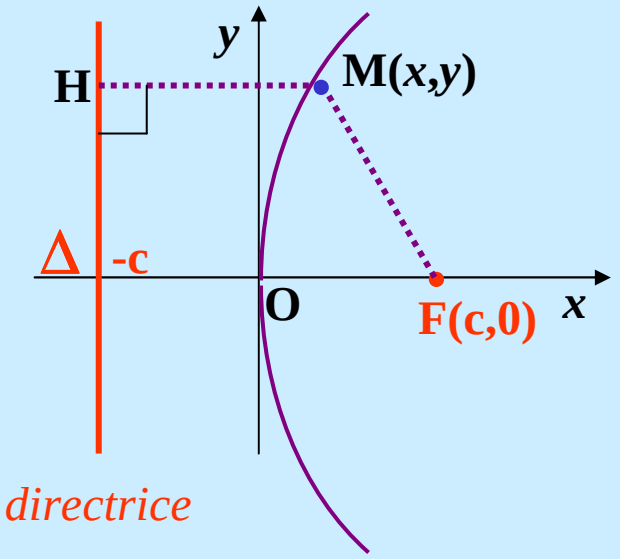
3. Equation cartésienne

parabole $MH = MF$

$$(x-(-c))^2 = (x-c)^2 + (y-0)^2$$

$$y^2 = (x+c)^2 - (x-c)^2$$

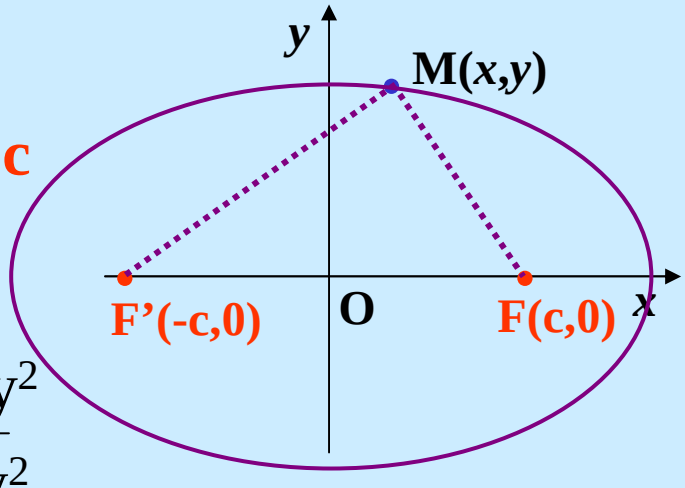
$$\boxed{y^2 = 4cx}$$



ellipse

$$MF' + MF = 2a$$

$$\sqrt{(x+c)^2 + y^2} + \sqrt{(x-c)^2 + y^2} = 2a \quad \textit{avec } a > c$$



$$2a - \sqrt{(x+c)^2 + y^2} = \sqrt{(x-c)^2 + y^2}$$

$$4a^2 - 4a\sqrt{(x+c)^2 + y^2} + (x+c)^2 + y^2 = (x-c)^2 + y^2$$

$$4a^2 + (x+c)^2 + y^2 - (x-c)^2 - y^2 = 4a\sqrt{(x+c)^2 + y^2}$$

$$4a^2 + 4cx = 4a\sqrt{(x+c)^2 + y^2} \qquad (a^2 + cx)^2 = a^2\,[(x+c)^2 + y^2\,]$$

$$a^4 + 2a^2cx + c^2x^2 = a^2x^2 + a^2c^2 + a^2y^2 + 2\,a^2cx$$

$$a^4 - a^2c^2 = a^2\,(a^2 - c^2) = (a^2 - c^2)x^2 + a^2y^2$$

posons : $a^2 - c^2 = b^2$

$$a^2b^2 = b^2x^2 + a^2y^2$$

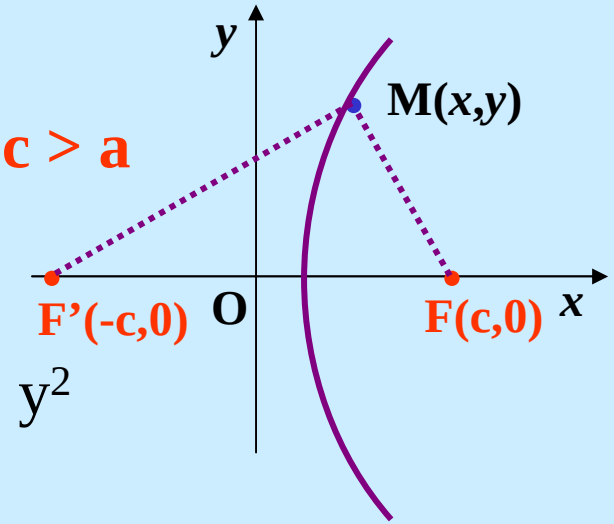
$$\boxed{x^2 / a^2 + y^2 / b^2 = 1}$$

hyperbole

$$| MF' - MF | = 2a$$

$$\sqrt{(x+c)^2 + y^2} - \sqrt{(x-c)^2 + y^2} = \pm 2a$$

$$\pm 2a - \sqrt{(x+c)^2 + y^2} = -\sqrt{(x-c)^2 + y^2}$$



avec **c > a**

$$4a^2 - (\pm)\, 4a\sqrt{(x+c)^2 + y^2} + (x+c)^2 + y^2 = (x-c)^2 + y^2$$

$$4a^2 + (x+c)^2 + y^2 - (x-c)^2 - y^2 = \pm 4a\sqrt{(x+c)^2 + y^2}$$

$$4a^2 + 4cx = \pm 4a\sqrt{(x+c)^2 + y^2} \qquad (a^2 + cx)^2 = a^2\,[(x+c)^2 + y^2\,]$$

$$a^4 + 2a^2cx + c^2x^2 = a^2x^2 + a^2c^2 + a^2y^2 + 2\,a^2cx$$

$$a^4 - a^2c^2 = a^2\,(a^2 - c^2) = (a^2 - c^2)x^2 + a^2y^2$$

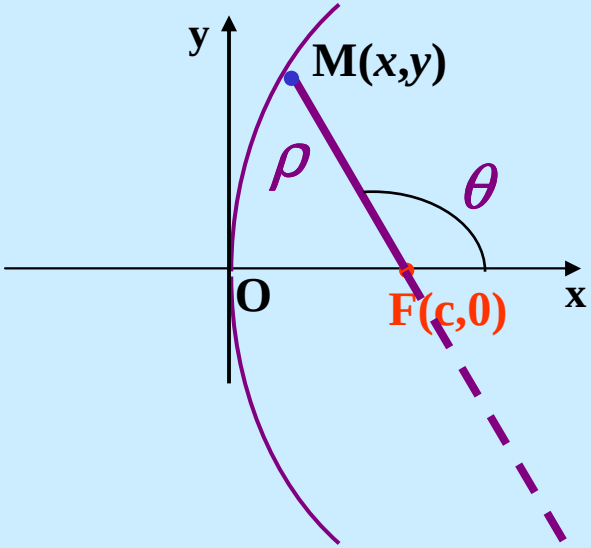
posons : $c^2 - a^2 = b^2$

$$a^2b^2 = b^2x^2 - a^2y^2$$

$$\boxed{x^2 / a^2 - y^2 / b^2 = 1}$$

4. Equation en coordonnées polaires

parabole $y^2 = 4cx$



parabole

$x = \rho \cos\theta + c$

$y = \rho \sin\theta$

$\rho^2 \sin^2\theta = 4c\,(\rho \cos\theta + c)$

$\rho^2 \sin^2\theta - 4c\,\rho \cos\theta - 4c^2 = 0$

$\rho^2 (1 - \cos^2\theta) - 4c\,\rho \cos\theta - 4c^2 = 0$

$\rho^2 (1 + \cos\theta)(1 - \cos\theta) - 4c\,\rho \cos\theta - 4c^2 = 0$

$[\rho (1 + \cos\theta) + 2c]\,[\rho (1 - \cos\theta) - 2c] = 0$

~~$\rho = -2c / (1 + \cos\theta)$~~ $\rho = 2c / (1 - \cos\theta)$

$\rho > 0$

$\rho \Leftrightarrow -\rho$ et $\theta \Leftrightarrow \theta + \pi$

ellipse

$$b^2 x^2 + a^2 y^2 = a^2 b^2$$

$$x = \rho \cos\theta + c$$

$$y = \rho \sin\theta$$

$$b^2 (\rho \cos\theta + c)^2 + a^2 (\rho \sin\theta)^2 = a^2 b^2$$

$$\rho^2 b^2 \cos^2\theta + 2 \rho b^2 c \cos\theta + b^2 c^2 + a^2 \rho^2 \sin^2\theta - a^2 b^2 = 0$$

$$\rho^2 [a^2 + (b^2 - a^2) \cos^2\theta] + 2 \rho b^2 c \cos\theta + b^2 (c^2 - a^2) = 0$$

posons : $a^2 - c^2 = b^2$

$$\rho^2 [a^2 - c^2 \cos^2\theta] + 2 \rho b^2 c \cos\theta - b^4 = 0$$

$$[\rho [a - c \cos\theta] + b^2] [\rho [a + c \cos\theta] - b^2] = 0$$

posons : $p = b^2 / a$ et $e = c / a$, $e < 1$

$$\rho = p / (1 + e \cos\theta)$$

~~$\rho = - p / (1 - e \cos\theta)$~~

$$\rho \Leftrightarrow - \rho \text{ et } \theta \Leftrightarrow \theta + \pi$$

$\rho > 0$

hyperbole

$$b^2 x^2 - a^2 y^2 = a^2 b^2$$

$$x = \rho \cos\theta + c$$

$$y = \rho \sin\theta$$

$$b^2 (\rho \cos\theta + c)^2 - a^2 (\rho \sin\theta)^2 = a^2 b^2$$

$$\rho^2 b^2 \cos^2\theta + 2 \rho b^2 c \cos\theta + b^2 c^2 - a^2 \rho^2 (1 - \cos^2\theta) - a^2 b^2 = 0$$

$$- \rho^2 [a^2 - (b^2 + a^2) \cos^2\theta] + 2 \rho b^2 c \cos\theta + b^2 (c^2 - a^2) = 0$$

posons : $c^2 - a^2 = b^2$

$$\rho^2 [a^2 - c^2 \cos^2\theta] - 2 \rho b^2 c \cos\theta - b^4 = 0$$

$$[\rho [a - c \cos\theta] + b^2] [\rho [a + c \cos\theta] - b^2] = 0$$

posons : $p = b^2 / a$ et $e = c / a$, $e > 1$

$$\rho = p / (1 + e \cos\theta)$$

~~$\rho = - p / (1 - e \cos\theta)$~~

$$\rho \Leftrightarrow - \rho \text{ et } \theta \Leftrightarrow \theta + \pi$$

$\rho > 0$